Achieving Energy-Efficiency in Two-Tiers Wireless Backhaul HetNets

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Abstract—We consider a model of two-tier heterogeneous cellular networks (HetNets), with wireless backhaul communication (WBC), consisting of a macrocell and a small cell tiers. A joint design of transmit beamforming, power allocation and bandwidth partitioning for both uplink (UL) and downlink (DL) transmissions is considered in this work. By proposing a strategy to partition the bandwidth for two consecutive time slots by two separate partitioning factors, we formulate a constrained optimization problem with the objective of maximizing the total energy efficiency of the small cells considering both UL and DL. For this non-convex problem, we leverage the sequential parametric convex approximation (SPCA) method to develop an efficient iterative algorithm to find the local optimal solution. Numerical simulations corroborate the convergence of our proposed algorithm and their performance gains compared to the previous work.

I. INTRODUCTION

Network densification is considered as one of the key factors for the thousand-fold increase in network capacity for next generation wireless networks [1]. According to [2] 80% of network energy is consumed at the base station (BS) sites; thus, saving energy on such large scale deployment of BSs simply translates to a greener and economical communication. Recently, wireless backhaul (WB) [3]–[5] emerged to present a simple and effective solution to the challenging problem of inconvenient and expensive installation. However, WBC should be able to guarantee the requirements of high throughput and reliable backhaul transmission to maintain a certain level of quality of service (QoS) in the network. In light of this, it is imperative to consider an energy efficient design for wireless backhaul networks.

In general, resource allocation has been optimized as an effective approach to obtain better network performance in the last few years [6], [7]. In [8], the authors reuse the concept of reverse time division duplexing (RTDD) [9] to consider the joint bandwidth allocation and user association that maximizes the achievable sum rate of small cells. The work in [10] studies the admission control of small cell access points (SAPs) in order to permit WB to serve as many SAPs as possible while guaranteeing predetermined QoS rates. Unlike [8]–[10], there is only one work in [11] that considers the WB network by making use of stochastic geometry theory and analyzing the energy efficiency (EE) of the small cell.

To the best of our knowledge, most existing works related to WBC focuses only on the resources that are allocated to the either UL or DL [8], [10]. In WB HetNets applying RTDD and spectrum splitting [8], the consideration of DL system design without including the relationship of backhaul rate and access rate on the UL might harm the UL access transmission quality. This is because the RTDD-spectrum splitting strategy has coupled the DL and UL. Thus, conventional approaches of one side transmission significantly degrade the performance on the other link and may cause the WBC on that link to be infeasible to support their access communication. This motivates the need for joint resource design on both UL and DL. Furthermore, there is no work that addresses the EE resource allocation in the WBH HetNets. Motivated from the above observations, this paper proposes a novel design of spectrum splitting that simultaneously accommodates both the WBC and wireless access communication (WAC) where the objective is to maximize the total EE of the small cells for both UL and DL. The formulated optimization problem is non-convex due to its natural property introduced by WBC. To solve this problem, we transform it into a tractable form and invoke the framework of sequential parametric convex approximation (SPCA) (introduced in [12]) to approximate the non-convex problem by its lower bound concave problem. Then, we propose an efficient iterative joint beamforming and resource allocation optimization (JBRAO) algorithm. Numerical studies confirm the convergence of our proposed iterative algorithm and their performance gains compared to the previous work.

The rest of the paper is organized as follows. Section II introduces the system model and derives the EE for the small cells. Section III formulates the optimization problem that maximizes the small cell EE on the UL and DL and proposes an iterative algorithm to solve the optimization problem. Section IV presents our numerical results under different simulation setups and discussions. Finally, conclusion of the paper is given in Section V.

II. SYSTEM MODEL

A. Spatial Model

This work considers a model of two-tier HetNets consisting of a MBS in the macrocell tier and F SAPs in the small cell tier. The MBS is equipped with N antennas to communicate...
with its $M$ macrocell users (MUEs) and SAPs. The SAPs and users are equipped with only one antenna. Even though we assume, for simplicity, that each SAP serves only one SUE, the general case of multiple SUEs in one small cell can be solved by following the same framework. We consider the operation of WBC of the SAPs on the UL and DL, where the SAPs are allowed to transmit and receive backhaul data concurrently with the macrocell on the same spectrum. We denote the conventional communication in this paper as WAC. Therefore, the communications in our model are categorized into two types: WBC between SAPs and MBS, and WAC between MBS (or SAPs) and MUEs (or SUEs), respectively.

B. Reverse time division duplex (RTDD)

We consider the RTDD operating mode as in [9]. Specifically, RTDD system reverses the UL/DL time slot in two-tiers so that when the MBS transmits a signal to its MUEs on the DL, each SUE transmits its signal to its serving SAP on the UL. A similar transmission protocol applies to the UL of the macrocell and DL of the small cell. However, when WBCs coexist with macrocell and small cell transmissions as in [9], assuming in-band half-duplex SAPs, an additional resource dimension such as frequency should be taken into account to avoid the self-interference arising at the SAPs. An example of time slot configuration is illustrated in Fig. 1.

C. Signal model

1) Macrocell DL - small cell UL: The channel is assumed flat over the spectrum and time-invariant within coherence time $T_c$ that is larger than the duration of two consecutive time slots. Thus, considering the time slot dedicated for macrocell DL and small cell UL transmissions, i.e., $T_{UL}$, the spectrum is assumed to be divided into $W$ resource blocks (RBs) of 1 Hz each. The $W$ RBs are split into two parts with splitting factor of $\alpha^d < 1$. On $[\alpha^dW]$ RBs, the MBS transmits data to its MUEs and the SAPs via WAC and WBC, respectively, while in the remaining $[(1-\alpha^d)W]$ RBs, each SUE simultaneously transmits data to its serving SAP. For convenience, we denote $G = \{ F, M \} = \{ \{ 1, \ldots, F \}, \{ F+1, \ldots, F+M \} \}$, where SAP indexes are from 1, and $F$ and MUE indexes are from $F+1, \ldots, F+M$. The received signal, within $T_{UL}$ in $[\alpha^dW]$ RBs, at the $j$th receiver is

$$ y_j = v_j^H h_j x_j + \sum_{k \neq j} v_k^H h_j x_k + n_j, \quad (1) $$

where $h_j \in \mathbb{C}^{N \times 1}$ is channel state vector which includes fading gain and pathloss components; $v_j \in \mathbb{C}^{N \times 1}$ denotes that beamforming vector from the MBS to the $j$th receiver; $x_j$ is the message intended for the $j$th receiver with unit average power and $n_j$ is the additive white Gaussian noise (AWGN) at the $j$th receiver, distributed according to normal distribution $CN(0, \sigma_0^2)$. We consider the general case of multiple SUEs in one small cell can be solved by following the same framework. We assume, for simplicity, that each SAP serves only one SUE, $\psi_i$ is the set of transmit powers of SUEs, $\mathbf{p}^u = [p_1^u, \ldots, p_F^u]^T$. By denoting $h_j^u$ as the scalar channel coefficient from the $i$th SAP to the SUE in the $j$th small cell which includes fading gain and pathloss components as $h_j^u = \rho_j^u \mathbf{h}_j^u + [\alpha^dW]\sigma_0$, the achievable rate at $i$th receiver, distributed according to normal distribution $CN(0, \sigma_0^2)$, where $\Gamma_i^u$ is the achievable rate at $i$th receiver calculated as

$$ \Gamma_i^u = \frac{\left| v_j^H h_j^u \right|^2}{\sum_{k \neq j} \left| v_k^H h_j^u \right|^2 + [\alpha^dW]\sigma_0}, \quad (2) $$

where the set of transmit beamforming at the MBS is denoted by $v \triangleq [v_1^T, \ldots, v_F^T, v_{F+1}^T, \ldots, v_{F+M}^T]$. Therefore, the communications in our model are categorized into two types: WBC between SAPs and MBS, and WAC between MBS (or SAPs) and MUEs (or SUEs), respectively.

2) Macrocell UL - small cell DL: In the time slot dedicated for macrocell UL and small cell DL, $T_{DL}$, $W$ RBs are also divided into two parts by a factor $\alpha^u < 1$, where in $[\alpha^uW]$ RBs, the MUEs and SAPs transmit signal to MBS through WAC and WBC, respectively, while in $[(1-\alpha^u)W]$ RBs, each SAP simultaneously transmits its signal to its intended MUE. By applying the above notation for the set of indexes at the SAPs and MUEs to denote the set of transmit power of the SAPs and MUEs as $\rho = \{ \rho_1, \ldots, \rho_F, \rho_F+1, \ldots, \rho_F+M \}$, we can write the received signal at the MBS within $T_{DL}$ period in $[\alpha^uW]$ RBs as

$$ y = \sum_{j=1}^{F+M} \mathbf{h}_j \sqrt{T_j} s_j + n, \quad (4) $$

where $\mathbf{n} \in \mathbb{C}^{N \times 1}$ is the AWGN vector at the MBS with distribution $CN(0, [\alpha^uW]\sigma_0^2)$, where $I$ is the $N \times N$ identity matrix. $s_j$ is the message from MUE or SAP with unit average power, e.g., $E \{ s_j s_j^H \} = 1$. The minimum mean square error (MMSE) beamforming $w_j = \left( \sum_{k \neq j} \rho_k \mathbf{h}_k \mathbf{h}_k^H + [\alpha^uW]\sigma_0 \right)^{-1} \mathbf{h}_j$ is applied at the MBS to detect transmitted signal from the $j$th SAPs and MUEs. Again, by treating interference as noise, the achievable rate at the $j$th user can be presented as in [13]

$$ R_j^u = [\alpha^uW] \log(1 + \Gamma_j^u) = [\alpha^uW] \log(1 + p_j^u \mathbf{h}_j^H \mathbf{h}_j^{-1} \mathbf{h}_j), \quad (5) $$

where $h_j \in \mathbb{C}^{N \times 1}$ is channel state vector which includes fading gain and pathloss components; $v_j \in \mathbb{C}^{N \times 1}$ denotes that beamforming vector from the MBS to the $j$th receiver; $x_j$ is the message intended for the $j$th receiver with unit average power and $n_j$ is the additive white Gaussian noise (AWGN) at the $j$th receiver, distributed according to normal distribution $CN(0, \sigma_0^2)$. We consider the general case of multiple SUEs in one small cell can be solved by following the same framework. We assume, for simplicity, that each SAP serves only one SUE, $\psi_i$ is the set of transmit powers of SUEs, $\mathbf{p}^u = [p_1^u, \ldots, p_F^u]^T$. By denoting $h_j^u$ as the scalar channel coefficient from the $i$th SAP to the SUE in the $j$th small cell which includes fading gain and pathloss components as $h_j^u = \rho_i^u \mathbf{h}_j^u + [\alpha^dW]\sigma_0$, the achievable rate at $i$th receiver calculated as

$$ \Gamma_i^u = \frac{\left| v_j^H h_j^u \right|^2}{\sum_{k \neq j} \left| v_k^H h_j^u \right|^2 + [\alpha^dW]\sigma_0}, \quad (2) $$

where the set of transmit beamforming at the MBS is denoted by $v \triangleq [v_1^T, \ldots, v_F^T, v_{F+1}^T, \ldots, v_{F+M}^T]$. Therefore, the communications in our model are categorized into two types: WBC between SAPs and MBS, and WAC between MBS (or SAPs) and MUEs (or SUEs), respectively.

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where $\Sigma_j = \sum_{k \neq j} \rho_k h_k h_k^H + [\alpha^W] \sigma_0 I$ is the covariance matrix, with $j \in G$. In the other $\{(1 - \alpha^W)\}$ RBs, each $i$th SAP transmits data to its intended SUE in the $i$th small cell with power $p_{d}^{i}$. By denoting $h_{dij}^{i} $ as the channel from the $i$th SAP to the SUE in the $j$th small cell and $p_{d}^{i} = [p_{d1}^{i}, \ldots, p_{dF}^{i}]^T$ as the set of transmit powers of the SAPs, the achievable rate at each small cell on the downlink can be written as $r_{d}^{i} = [(1 - \alpha^W)] \log (1 + \gamma_{d}^{i})$ where $\gamma_{d}^{i}$ is given by

$$
\gamma_{d}^{i} = \frac{p_{d}^{i} |h_{dij}^{i}|^2}{\sum_{j \neq i} p_{d}^{j} |h_{dij}^{j}|^2 + [(1 - \alpha^W)] \sigma_0}.
$$

(D. Small Cell Energy Efficiency (SMEE))

To express the total EE, calculated in (nats/Joule), we adopt the linear power consumption model where the total small cell power consumption is given by

$$
P = \sum_{i=1}^{F} (\kappa_{d} p_{d}^{i} + \kappa_{u} p_{u}^{i}) + P_0,
$$

where $P_0$ stands for the constant value that is independent of transmit power $p_{d}^{i}$ and $p_{u}^{i}$. Specifically, this metric consists of the total circuit power, including the power dissipation in the transmitting filter, mixer, frequency synthesizer and digital-to-analog converter at all the SAPs and SUEs. In addition, $\kappa_{d}, \kappa_{u} > 1$ are the constant which accounts for the inefficiency of the power amplifier at each SAP and SUE when they transmit on the DL and UL in WAC. Now, we can compute the SMEE on the DL and UL as

$$
\theta = \frac{\sum_{i=1}^{F} (r_{d}^{i} + r_{u}^{i})}{\eta}.
$$

(III. SMEE Optimization Problem)

(A. Problem formulation)

We formulate a problem to jointly optimize the beamforming, power allocation together with fraction of bandwidth usage $\alpha^d$ and $\alpha^u$ to maximize the total SMEE defined in (8). Denoting $p = [p_{d}^T, p_{u}^T]^T$ as the set of transmit power at the SUEs and SAPs in the WAC, the SMEE optimization problem can be casted as:

$$
\max_{\alpha^d, \alpha^u, p} \theta
$$

s.t. $r_{d}^{i} \geq r_{d}^{i}_{\min}, \forall i \in F$, $\gamma_{d}^{i} \geq 1, \forall j \in M$, $\sum_{k=1}^{F+M} \|v_{k}\|^2 \leq P_m$, $0 \leq p_{d}^{i} \leq P_{d}, \forall i \in F$, $0 \leq p_{u}^{i} \leq P_{u}, \forall i \in F$, $\forall i \in A = \{d, u\}$,

where the objective function (9a) is the total EE of all the small cells on the DL and UL. The constraint (9b) imposes that the backhaul achievable rate of each SAP via WBC should always be larger than the corresponding achievable rate at each SAP via WAC on the UL and DL, respectively. On the other hand, the achievable rates at the MUEs via WAC are regulated by (9c) to be larger than minimum rate requirements $r_{d}^{i}_{\min}$. (9d)-(9f) are the constraints accounts for the maximum power at the MBS, SAP, MUE, and SUE. Note that the difference between the formulated problem in (9) and the conventional MU-MISO problem lies in (9b) where the lower bound of the achievable rate at each SAP is not fixed as in (9c).

Solving problem (9) with the ceiling $\lceil \cdot \rceil$ and flooring $\lfloor \cdot \rfloor$ functions is difficult. Thus, we propose to remove these functions and solve the relaxed problem with respect to all involved variables. The solution of the relaxed problem is finally recovered by rounding up or down accordingly. Observing that the problem at hand is complex and intractable in its current form, our approach to solve problem (9) constitutes of firstly transforming it into equivalent form with more tractability and secondly by appropriate approximations.

(B. Subtractive equivalent form)

The objective of (9) is in a fractional form, which is intractable in general. Therefore, we transform it into the subtractive form to aid the algorithm development. Toward this end, let us denote $\Omega$ as the set of feasible solutions $\alpha, v, \rho, p$ of (9), we can express the maximum $\theta^{*}$ as

$$
\theta^{*} = \max_{(\alpha, v, \rho, p) \in \Omega} \frac{\sum_{i=1}^{F} (r_{d}^{i} + r_{u}^{i})}{P^{*}} = \frac{\max_{(\alpha, v, \rho, p) \in \Omega} \sum_{i=1}^{F} (r_{d}^{i} + r_{u}^{i})}{P^{*}},
$$

where $r_{d}^{i}, r_{u}^{i}, \rho^{*}$, $P^{*}$ are the values of the DL rate, UL rate and consumed power evaluated at the optimal solution. According to [14], for $r_{d}^{i}, r_{u}^{i} \geq 0$ and $P > 0$, the maximum SMEE $\theta^{*}$ defined in (10) is achieved if and only if $F(\theta^{*}) = \max_{(\alpha, v, \rho, p) \in \Omega} \sum_{i=1}^{F} (r_{d}^{i} + r_{u}^{i}) - \theta^{*} P = \sum_{i=1}^{F} (r_{d}^{i} + r_{u}^{i}) - \theta^{*} P^{*}$. This result imply that the optimal value of $\theta^{*} \geq 0$ must satisfy $F(\theta^{*}) = 0$. Thus to solve (9), we can consider iteratively solving (11) for a current value of $\theta$ and updating until we reach the optimal $\theta^{*} \geq 0$ satisfying $F(\theta^{*}) = 0$.

$$
\max_{(\alpha, v, \rho, p) \in \Omega} \left\{ \sum_{i=1}^{F} (r_{d}^{i} + r_{u}^{i}) - \theta P(9b) - (9g) \right\}.
$$

(C. Equivalent transformations)

The problem (11) is still complicated due to its non-convex property underlying in the objective and constraints. We first deal with the non-concave objective function of (11). By introducing the slack variables $q_{d}^{i}, q_{u}^{i} \geq 0$, where $i \in F$, we can reformulate (11) as

$$
\max_{(\alpha, v, \rho, p, q, q) \geq 0} \left\{ \sum_{i=1}^{F} (W q_{d}^{i} + W q_{u}^{i}) - \theta P(9b) - (9g), \right\}
$$

$$
\gamma_{d}^{i} \geq e^{q_{d}^{i}/(1 - \alpha^{A\{d\}) - 1}, \forall i \in F,}
$$

where $q = [q_{d}^{1}, \ldots, q_{d}^{F}, q_{u}^{1}, \ldots, q_{u}^{F}]^T$ are the new variables. By multiplying the term $(1 - \alpha^{A\{d\}})$ on both sides of the new
constraint, we can further decompose this constraint into two new inequalities with the help of new slack variables \( t = [t_1^u, \ldots, t_F^u, t_1^d, \ldots, t_F^d]^T \), where \( t_i^u, t_i^d \geq 0, \forall i \in \mathcal{F} \), as

\[
(1 - \alpha^{A(o)}) \gamma_i^u \geq t_i^u, \forall i \in \mathcal{F}, \quad (13a)
\]

\[
(1 - \alpha^{A(o)}) \left( e^{a_i^u^n/(1 - \alpha^{A(o)})} - 1 \right) \leq t_i^u, \forall i \in \mathcal{F}. \quad (13b)
\]

We observe that (12) with additional variables \( t \) and (13) are equivalent since at optimality, all the constraints in (13b) are active. Next, to further decompose (13a) into more tractable form, we rewrite (13a) as

\[
(1 - \alpha^{A(o)}) p_i^o |h_{i,i}|^2 \geq t_i^o z_i^o, \forall i \in \mathcal{F}, \quad (14a)
\]

\[
(1 - \alpha^{A(o)}) W \sigma_0 \leq z_i^o, \forall i \in \mathcal{F}, \quad (14b)
\]

where \( z_i^o > 0 \) and \( z_i^u > 0 \) are the newly introduced variables, and we denote \( z = [z_1^o, \ldots, z_F^o, z_1^u, \ldots, z_F^u]^T \). Now, we rewrite each inequality in (9b) as

\[
\alpha o_i a_i^o \geq (1 - \alpha^{A(o)}) b_i^o, \forall i \in \mathcal{F}, \quad (15a)
\]

\[
a_i^o \leq \log (1 + \Gamma_i^o), \forall i \in \mathcal{F}, \quad (15b)
\]

\[
b_i^o \geq \log (1 + \gamma_i^o), \forall i \in \mathcal{F}, \quad (15c)
\]

where \( a_i^o \geq 0, b_i^o \geq 0 \) and \( a_i^o \geq 0, b_i^o \geq 0 \) are the newly introduced variables we denote \( a = [a_1^o, \ldots, a_F^o, a_1^n, \ldots, a_F^n]^T \) and \( b = [b_1^o, \ldots, b_F^o, b_1^n, \ldots, b_F^n]^T \) for later usage. At this point, we can reformulate (12) into a new equivalent form as:

\[
\max_{\alpha, \eta, \nu, \rho, p, h, \sigma, \theta} \sum_{i=1}^{N} (W_i^u + W_i^d) - \theta P \quad (16a)
\]

\[
\text{subject to:} \quad (13b), (14) - (15), \quad (16b)
\]

\[
\sum_{k \neq i} |v_k^H h_i|^2 / \nu_i \geq e_i^o - 1, \forall \nu_i \in \mathcal{G}, \quad (16c)
\]

\[
\sum_{k \neq i} |v_k^H h_i|^2 + \alpha o W \sigma_0 \leq \nu_i, \forall \nu_i \in \mathcal{G}, \quad (16d)
\]

\[
\eta_i^2 h_i^H \Sigma_i^{-1} h_i \geq e_i^o - 1, \forall \nu_i \in \mathcal{G}, \quad (16e)
\]

\[
\rho_i \geq e_i^o, \forall \nu_i \in \mathcal{G}, \quad (16f)
\]

\[
a_i^o \geq \gamma_i^o / (W \sigma_0), \forall j \in \mathcal{M}, \quad (16g)
\]

\[
(9d) - (9g), \quad (16h)
\]

where the constraints in (16c) and (16d) are the results from the equivalent decomposition from (15b) and (9c) with \( o = d \). Note that in (16), we have introduced a set of new variables \( \nu = [\nu_1, \ldots, \nu_{F+M}]^T \) with additional constraints (16c) and (16d). To leverage the presentation of all the constraints in (15b) and (9c) in a unified manner, we have introduced additional the constraint (16g) to exhibit the similarity between (15b) and (9c). Similarly, we rewrite (15b) and (9c) with \( o = u \) by (16e) and (16f), respectively, where \( \eta_i, \forall i \in \mathcal{F} \) are the newly introduced variables and \( \eta = [\eta_1, \ldots, \eta_F, \eta_{F+1}, \ldots, \eta_{F+M}]^T \). The reason for introducing \( \eta \) will be shown shortly in the next subsection as it is necessary for the usage of semi-definite programming (SDP) expression.

### D. Problem approximations

Note that (16) is still non-convex due to the existence of non-convex constraints, except for (13b), (14b), (16d), (16f), (16g) and (9d)-(9f). Note that we rely on the result that presents the three conditions in [15] which can be employed to verify the proposed convex approximations. The proposed convex approximation should meet these conditions so that the developed iterative algorithm based on that will converge to the solution that satisfies the KKT conditions of problem (12). In general, each non-convex function can be replaced by an upper bound approximated function of the same variable with some newly introduced constants. We briefly clarify the correspondence between the non-convex constraints and their approximation as follow.

In (16), we apply the convex approximation [12] for the right side of (14a), (15a) then apply simple manipulations to rewrite them in second order cone (SOC) constraint as in (18b) and (18c). Details of the approximation are omitted for brevity. Next, we observe that (16c) and (16e) contain the different of convex (DC) form due to the convex factor on their left side. Thus, we apply the first order Taylor’s approximation on these terms to obtain (18d) where

\[
F \left( h_i^{(n)}, v_i^{(n)}, v_i^{(n)}, \nu_i^{(n)}, \nu_i^{(n)} = H \nu_i^{(n)} \right) = \mathcal{B} \left( \nu_i^{(n)} \right) \quad (16a)
\]

\[
p_i^o |h_{ij}|^2 + \left( 1 - \alpha^{A(o)} \right) W \sigma_0 \leq \nu_i^{(n)}, \forall i \in \mathcal{F}, \quad (16b)
\]

\[
\sum_{j \neq i} \left( \nu_i^{(n)} + \eta_i^{(n)} h_{ij}^H \nu_i^{(n)} - \nu_i^{(n)} - \nu_i^{(n)} h_{ij}^H \nu_i^{(n)} \right), \quad (16c)
\]

\[
\nu_i^{(n)} = H \left( \eta_i^{(n)} \nu_i^{(n)}, \nu_i^{(n)} \right) = - \left( \eta_i^{(n)} \nu_i^{(n)} \right) + \left( \nu_i^{(n)} \right)^2, \quad (16d)
\]

where

\[
(16e)
\]

\[
\sum_{j \neq i} \left( \nu_i^{(n)} + \eta_i^{(n)} h_{ij}^H \nu_i^{(n)} - \nu_i^{(n)} - \nu_i^{(n)} h_{ij}^H \nu_i^{(n)} \right), \quad (16f)
\]

\[
\nu_i^{(n)} = H \left( \nu_i^{(n)}, \nu_i^{(n)} \right) = - \left( \nu_i^{(n)} \right), \quad (16g)
\]

\[
\sum_{j \neq i} \left( \nu_i^{(n)} + \eta_i^{(n)} h_{ij}^H \nu_i^{(n)} - \nu_i^{(n)} - \nu_i^{(n)} h_{ij}^H \nu_i^{(n)} \right), \quad (16h)
\]

where we can see that the non-convex property of this constraint lies in the right side of (17). Similarly, we apply the first order Taylor’s approximation on this term to obtain (18e), where

\[
G_i^{(n)} \left( p_i^{(n)}, 1 - \alpha^{A(o)} \right) = \log \left( g_i^{(n)} \left( p_i^{(n)}, \alpha^{A(o)} \right) \right) + \frac{g_i^{(n)} e_i^{(n)}}{g_i^{(n)} e_i^{(n)}} \cdot \eta_i^{(n)} \left( \nu_i^{(n)} \right) \nu_i^{(n)} \right) + \left( 1 - \alpha^{A(o)} \right) W \sigma_0 \right) \right), \quad (17a)
\]

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\[
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\]

where we can see that the non-convex property of this constraint lies in the right side of (17). Similarly, we apply the first order Taylor’s approximation on this term to obtain (18e), where
Note that \( \alpha^{(n)}_i, \alpha^{u(n)}_i, \xi^{(n)}_i, \psi^{o(n)}_i, P_i^{o(n)} \) for \( i \in F, o \in A \) and \( q_{(n)}^{(l)}, \rho_{(n)}^{(l)}, v_{(n)}^{(l)}, \nu_{(n)}^{(l)} \) for \( l \in G \) are not the variables of the optimization problem but the parameters that is iteratively updated by the optimum solution after each iteration. For brevity, we denote \( \xi^{(n)} = [\xi^{(n)}_1, \ldots, \xi^{(n)}_F] \), \( \psi^{o(n)} = [\psi^{o(n)}_1, \ldots, \psi^{o(n)}_F] \), \( \eta^{(n)} = [\eta^{(n)}_1, \ldots, \eta^{(n)}_F] \) and \( \nu^{(n)} = [\nu^{(n)}_1, \ldots, \nu^{(n)}_F] \). The pseudo code presents the JBRAO Algorithm to solve the optimization problem is summarized in the Algorithm 1. The convergence of Algorithm 1 can be proved similarly to [12], and the proof is omitted here for brevity.

\[
\max_{\alpha_i, \xi_i \geq 0, \eta_i \geq 0} \sum_{i=1}^{F} \left( W_i q_i^u + W_i q_i^d \right) - \theta P
\]

\[
\text{s.t.} \quad \left\| \begin{array}{c} \frac{c_i^{(n)}}{2} t_i^c \left( 1 - \alpha^{(n)}_i - p_i^{(n)} \right) \sum_{l=1}^{G} \frac{1}{L^2 c_l^{(n)}} |h_{il}|^2 \left( 1 - \alpha^{(n)}_i - p_i^{(n)} \right) \right. \\
\left. \leq \frac{1 - \alpha^{(n)}_i + p_i^{(n)}}{2}, \forall i \in F, \right. \\
\left. \frac{1 - \alpha^{(n)}_i + p_i^{(n)}}{2}, \forall i \in F, \right. \\
F \left( h_i, v_i, v_i^{(n)}, v_i^{(n)} \right) \geq c_i^{(n)} - 1, \forall l \in G, \\
b_i^l + \log \left( \sum_{j=1}^{F} p_j^l |h_{ij}^l|^2 + (1 - \alpha^{(n)}_i) W \sigma_0 \right) \geq 0, \\
G_i^o \left( \rho_i^o, 1 - \alpha^{(n)}_i, \psi^{o(n)}_i \right) \leq 0, \forall i \in F, \\
H \left( \eta_i, \rho_i, \nu_i, \nu_i \right) \leq 0, \forall i \in G, \\
123456789,
\right.
\]

Algorithm 1: Iterative JBRAO Algorithm

1: Initial \( \theta^{(0)} = 0 \), tolerant factor \( \epsilon = 10^{-3} \);
2: \( t := 0 \);
3: repeat
4: Set \( t := t + 1 \);
5: Initialize \( \alpha^{(n)}_i, \xi^{(n)}_i, \psi^{o(n)}_i, \eta^{(n)}_i, \rho^{(n)}_i, \psi^{o(n)}_i, \psi^{o(n)}_i, \nu^{(n)}_i \);
6: Set \( n := 0 \);
7: repeat
8: Solve the convex problem in (18) to achieve the optimal solution \( \alpha^*, q^*, t^*, z^*, \alpha^*, b^*, v^*, \nu^*, \rho^*, P^*, \eta^* \);
9: Set \( n := n + 1 \);
10: Update \( \alpha^{(n)}_i = \alpha^*, \psi^{o(n)}_i = \psi^{o(n)}_i, \psi^{o(n)}_i = \psi^{o(n)}_i, \rho^{(n)}_i = \rho^*, \eta^{(n)}_i = \eta^*, \eta^{(n)}_i = \eta^*, \eta^{(n)}_i = \eta^* \);
11: until Convergence of the objective in (18);
12: Compute
\[
\theta^{(t)} = \frac{\sum_{i=1}^{F} \left( W_i q_i^{d(n)} + W_i q_i^{u(n)} \right)}{\sum_{i=1}^{P} \left( \kappa^{d}_i P_i^{d(n)} + \kappa^{u}_i P_i^{u(n)} \right) + P_b} \]

13: until \( |\theta^{(t)} - \theta^{(t-1)}| \leq \epsilon \);

IV. Numerical Results

In this section, we evaluate the performance of the JBRAO algorithm. We assume time-invariant and flat Rayleigh fading channels and the pathloss component is calculated as \( (d_{ij}/d_0)^{-3.8} \), where \( d_{ij} \) is the distance between the \( i \)th transmitter and the \( j \)th receiver, and \( d_0 = 100 \) m is the reference distance. We apply a circular coverage of macrocell with radius \( 10d_0 \). The MBS is positioned at the center, where there are \( F = 2 \) small cells evenly placed \( 7d_0 \) apart from the center. We assume that each small cell has a circular coverage of radius \( d_0 \) with its SAP at the center and a SUE at the circumference of each small cell coverage. In addition, we assume \( M = 2 \) MUEs uniformly scattered across the macrocell coverage. Unless being mentioned elsewhere, we choose this scenario as the standard model for the numerical simulation, where the number of transmit antenna as \( N = 4 \) and the maximum transmit power as \( P_m = 50 \) dBm, while at small cell, we choose the maximum transmit power at the SAP \( \tilde{p} = 35 \) dBm. To protect the QoS at each MUE, we choose the minimum rate requirement \( r_{\min} = 10^6 \) nats/s. In additions, we choose the maximum transmit power at each user as \( \tilde{p} = 35 \) dBm. Finally, the noise power is set \( W \sigma_0 = -120 \) dB and the bandwidth is \( W = 10 \) MHz.

In Fig. 2, we show the convergence of the ratio between the objective function in (9a) and the spectrum \( W \) in three schemes: (i) proposed scheme where the objective function of (9) contains two terms of UL and DL; (ii) UL scheme where (9a) only has one term of UL on the numerator and denominator; (iii) DL Scheme where (9a) has one term of downlink on the numerator and denominator. We compare the performance of three schemes by evaluating the SMEE with the achieved solutions. We observed that each SMEE corresponding to each scheme keeps increasing and converges after a few iterations \( t \), which validates the convergence property of Algorithm 1 discussed above. In addition, these results show the significant effectiveness of our proposed design compared to the traditional perspective of one side transmission design [8].

In Fig. 3, we show the SMEE on both the UL and DL.
transmission versus $\bar{p}_f^d$. The performance is compared at different values of $r_{\text{min}}^0 = 2, 4, 6, 8$ and $\times 10^5$ nats/s. We can observe that the SMEE increases with $\bar{p}_f^d$ and finally saturates at high value of $\bar{p}_f^d$, e.g., at $\bar{p}_f^d = 16$ dBm. This is obvious because when SAPs have higher individual power budget, they will choose to transmit at higher power to achieve more throughput. However, at higher level of $\bar{p}_f^d$, the SMEE tends to saturate since employing more power to achieve higher rate might degrade the overall energy efficiency. Thus, limited usage of transmit power is necessarily required to maintain the maximum SMEE. In addition, when $r_{\text{min}}^0$ increases, SMEE decreases accordingly. This can be explained as when the MUEs have higher minimum rate requirement for the QoS quality, more spectrum should be drawn towards the macrocell access transmission to guarantee the constraints. Thus, the radio resource saved for the small cell access link becomes less, which results in lower SMEE at the small cells.

In Fig. 4, we show the level of bandwidth partitioning dedicated for the DL access transmission from the SAPs to SUEs, namely $1 - \alpha^u$ when $P_m$ increases. The results are compared between the proposed, DL and UL schemes. We observe that when $P_m$ increases, the term $1 - \alpha^u$ for the proposed and DL scheme increases, while it remains constant for the UL scheme. This can be explained as $P_m$ only takes effect on the problem of maximizing the DL transmissions. When $P_m$ increases, more power budget is available to support the WBC on the DL. This in turn creates more resource for the DL, thus enforces the DL access transmissions to use more bandwidth to increase their achievable rates. This results in the increment of the term $1 - \alpha^u$ in the proposed and DL scheme.

V. CONCLUSION

This paper studies the joint design of transmit beamforming, power allocation and bandwidth partitioning by maximizing the total EE of the small cells in the UL and DL transmission in the two-tier HetNets using WBC. Employing the RTDD mode, we propose a strategy to partition the bandwidth two consecutive time slot by two different level and formulate a constrained optimization problem based on the proposed scheme. To overcome the obstacle of non-convexity, we develop an efficient iterative algorithm where in each iteration, we approximate the optimization problem by the SPCA method to find the local optimal solution at convergence. Numerical simulations are conducted to confirm our algorithm performance compared to the previous work.

REFERENCES