

MGL7160: Exercice #2

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a) Soit $e1$: $\text{set}\{\text{sequence}\{\text{nat}\}\}$ et $e2$: $\text{set}\{\text{sequence}\{\text{nat}\}\}$ les deux ensembles suivants dont les éléments sont des séquences de nombres naturels:

```
e1 = {[2..1], [1], [3..4], [1..4]}
e2 = {[], [1, 1, 0], [2..4], [0, 2], [0, 0]}
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1. $[3, 4] \text{ IN } \text{add}([3], e1) = \text{true}$
2. $\text{size}(\{[]\} \cup \text{add}([1..4], \text{add}([1, 4], e1))) = 5$
3. $(\{[1..0]\} \cup e2) - \{s: \text{sequence}\{\text{nat}\} \text{ SUCH THAT } 1 \text{ IN } s :: s\} = \{[], [2..4], [0, 2], [0, 0]\}$
4. $e1 - \{s: \text{sequence}\{\text{nat}\} \text{ SUCH THAT } s \text{ IN } e1 :: \text{add}(1, s)\} = \{[], [3..4], [1..4]\}$
5. $\{[1, 2, 3], \$e1\} - \{x: \text{nat} \text{ SUCH THAT } 1 \leq x \leq 2 :: [x, x+1, x+2]\} = e1$
6. $\text{UNION}(s: \text{sequence}\{\text{nat}\} \text{ SUCH THAT } s \text{ IN } e2 \ \& \ \text{length}(s) \leq 2 :: \{s\}) = \{[], [0, 2], [0, 0]\}$
7. $\text{INTERSECTION}(n: \text{nat} \text{ SUCH THAT } n \leq 4 \ \& \ [n..4] \text{ IN } e1 :: \text{add}(2, \{n\})) = \{2\}$

b) Soit $s1$: $\text{sequence}\{\text{set}\{\text{nat}\}\}$ et $s2$: $\text{sequence}\{\text{set}\{\text{nat}\}\}$ les deux séquences suivantes dont les éléments sont des ensembles de nombres naturels:

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s1 = {[3..4], {1}, {2..1}, {2..2]}
s2 = {[1..2], {3, 3}, {1, 2]}
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1. $\text{length}(\text{add}(\{2\}, [\{2\}, \$s1])) = 6$
2. $(s1 \parallel s2 \parallel s1)[6] = \{3\}$
3. $\text{remove}(\{n: \text{nat} \text{ SUCH THAT } n < 0 :: n\}, s1) = \{[3..4], \{1\}, \{2\}\}$
4. $\{e: \text{set}\{\text{nat}\} \text{ SUCH THAT } e \text{ IN } s2 \ \& \ \text{size}(e) \leq 1 :: e\} = \{\{3\}\}$
5. $\text{SUM}(n: \text{nat} \text{ SUCH THAT } n+1 \text{ IN } \text{domain}(s2) :: 2*n) = 2*0 + 2*1 + 2*2 = 6$
6. $\text{PRODUCT}(n: \text{nat} \text{ SUCH THAT } \{2..n\} \text{ IN } s1 :: 2*n+3) = (2*0+3) * (2*1+3) * (2*2+3) = 3 * 5 * 7 = 105$
7. $\text{NUMBER}(e: \text{set}\{\text{nat}\} \text{ SUCH THAT } \text{add}(2, e) \text{ IN } s1 :: e) = 2$

c) Soit $d1$: $\text{map}\{\text{string}, \text{sequence}\{\text{nat}\}\}$ et $d2$: $\text{map}\{\text{string}, \text{sequence}\{\text{nat}\}\}$ deux dictionnaires dont les clés sont des chaînes de caractères et les définitions des séquences d'entiers naturels et tels que:

```
d1 = {[ "", [5] ], [ "bcd", [1..6] ], [ "zzz", [2] ]; [] }
d2 = {[ "ab", [4, 4] ], [ "xyz", [1, 2] ]; [] }
```

1. $\text{range}(d1) = \{[], [2], [5], [1..6]\}$
2. $\text{remove}(\text{"ghj"}, \text{bind}(\text{"ghj"}, [7..6], d1))[\text{"ghj"}] = []$
3. $\text{bind}(\text{"uvw"}, [11], \text{bind}(\text{"xx"}, [2,2,2], \text{create}([11]))) = \{[\text{"xx"}, [2,2,2]]; [11]\}$
4. $\text{PRODUCT}(s: \text{string} \text{ SUCH THAT } 5 \text{ IN } d1[s] :: \text{length}(d1[s])) = \text{length}(d1[""]) * \text{length}(d1["bcd"]) = 1 * 6 = 6$
5. $\text{SUM}(s: \text{sequence}\{\text{nat}\} \text{ SUCH THAT } s \text{ IN } \text{range}(d2) - \{\text{default}(d2)\} :: 3*\text{length}(s)) = 3*2 + 3*2 = 12$
6. $\text{UNION}(n: \text{nat} \text{ SUCH THAT } [n-1] \text{ IN } \text{range}(d1) \mid n = 3 :: \{n, n+10\}) = \{6, 16\} \cup \{3, 13\} = \{3, 6, 13, 16\}$