Abstract

We introduce a logical formalism for describing properties of configurations of computing systems. This logic of trees allows quantification on nodes labels which are modalities containing variables. We explain the motivation behind our formalism and give a classical semantics and also a new equivalent one based on partial functions on variables.

1. Introduction

Managing computing equipment configurations is a central task in computer science. The increase in computational power and in number of computing devices makes this task error prone. In order to help automatize the management of devices configuration, it is important to be able to describe properties in a abstract way.

We propose in this paper a simple extension to modal logic, aimed at describing properties of computing equipments. Even if our motivation are towards applications in computer networks, our logical system is applicable to any type of devices.

Our objective is to develop a formalism to check, analyze and help debug complex configurations. In this paper we make a first step in this direction presenting the logical system, a classical semantics and also a new formal semantics in terms of partial functions on variables.

We have tried to keep our logical system as simple as possible and as close as possible to the objective of analyzing configuration.

The paper is structured as follows. We define our logic in section 2, we give examples in the field of computer networks in section 3, we relate our logic to others computational logic in section 4. Finally we develop the new formal semantics in section 5 and we conclude with section 6.

2. Configuration logic

2.1. Motivation

The goal behind the development of Configuration Logic is to describe and verify properties on configurations of computing equipment. In particular, we are interested in validating the configuration of network routers, which are the equipment responsible for forwarding packets towards their destination.

The configuration of a router is the set of parameter-value pairs that describe the state of the device at a given moment. These parameter-value pairs are organized in a hierarchic fashion: for example, each router may have multiple interfaces, and each interface has its IP address. Figure 1 shows the configuration of a router containing two interfaces, called eth0 and eth1, whose IP addresses are respectively 192.168.1.13 and 192.168.1.14.

In a parameter-value pair, the parameter is a static name, while the value is configurable. It is important to note that
the parameter-value pair is unique amongst its sibling. In our example the node \texttt{inter=eth0} represents the unique interface with name \texttt{eth0}.

The objectives of Configuration Logic are to describe properties of configurations, automate verifications and eventually to have a setting to generate configurations satisfying some property.

2.2. Syntax

A Configuration Logic language CL is formed of a set of names \( \text{Names} = \{ p, q, r, p_1, q_1, r_1, \ldots \} \), a set of variables \( \text{Variables} = \{ x, y, z, x_1, y_1, z_1, \ldots \} \) and a set of relations \( R_1(x), R_2(z), \ldots \) (respectively of arity \( \text{arity}(R_1), \text{arity}(R_2), \ldots \)).

Formulas are build-up using the usual boolean connectives \( \land, \lor, \neg \) and also the following quantifiers.

**Existential quantifiers:** There are two forms of existential quantifiers \( (p = x)\varphi \) and \( (\bar{p} = \bar{x}; p = x)\varphi \), where \( p \) is a name, \( \bar{p} \) a finite sequence of names, \( x \) a variable and \( \bar{x} \) a finite sequence of variables of the same length as \( \bar{p} \). Here only the last variable \( x \) is considered bound as will become clear in the classical semantics below.

**Universal quantifiers:** There are also two forms of universal quantifiers \( [p = x]\varphi \) and \( [\bar{p} = \bar{x}; p = x]\varphi \), where \( p, \bar{p}, x \) and \( \bar{x} \) are the same as for existential quantifiers. Here again only the last variable \( x \) is considered bound.

If it is necessary to explicitly write the elements \( p_1, \ldots, p_n \) of \( \bar{p} \) and those \( x_1, \ldots, x_n \) of \( \bar{x} \), we will write the quantifiers as \( (p_1 = x_1, \ldots, p_n = x_n; p = x)\varphi \) and \( [p_1 = x_1, \ldots, p_n = x_n; p = x]\varphi \).

In order to simplify proofs and definitions, we will consider that \( (p = x)\varphi \) and \( [p = x]\varphi \) are special cases of \( (\bar{p} = \bar{x}; p = x)\varphi \) and \( [\bar{p} = \bar{x}; p = x]\varphi \), where \( \bar{p} \) and \( \bar{x} \) are empty.

Furthermore we will, without loss of generality, restrict ourselves to sentences in which every variable is bounded only once. By renaming, every sentence can be put in this form.

In fact we want to limit the use of quantifiers in such a way that they are extensions of previous ones. To make this notion clear let us introduce the following definition.

**Definition 1.** A sentence is a formula such that every variable is bounded and furthermore any sub-formula

\[
(p_1 = x_1, \ldots, p_n = x_n; p = x)\psi
\]

is contained in a sub-formula which is not binding any \( x_1, \ldots, x_{n-1} \) and is of the form

\[
(p_1 = x_1, \ldots, p_{n-1} = x_{n-1}; p_n = x_n)\varphi
\]

or of the form

\[
[p_1 = x_1, \ldots, p_{n-1} = x_{n-1}; p_n = x_n]\varphi
\]

Similarly for sub-formulas \( [p_1 = x_1, \ldots, p_n = x_n; p = x]\psi \).

We will introduce some definitions before giving the classical semantics.

**Definition 2.** A path is a finite non-empty word on the alphabet formed of all \( \{ p = x \} \) where \( p \) is a name and \( x \) a variable.

**Definition 3.** A name-path is a finite non-empty word on the alphabet formed of all names.

If \( \bar{p} = p_1 \ldots p_n \) and \( \bar{x} = x_1 \ldots x_n \) we will usually write \( (\bar{p} = \bar{x}) \) for the path \( (p_1 = x_1) \cdots (p_n = x_n) \).

2.3. Configurations

A configuration is a forest (set of trees) such that every node is labeled by a name and a value. Furthermore there is no two roots (top level nodes) having the same name and value; similarly, every node has no more than one child having the same name and value. Formally we introduce the following definition.

**Definition 4.** A configuration is a structure of the form \( (V, N, \bar{R}_1, \ldots, \bar{R}_n) \) where:

- \( V \) is a set, whose elements are called values
- \( N \) is a set of words closed under prefix, on the alphabet formed of \( \{ p = v \} \), with \( p \) a name and \( v \in V \). The elements of \( N \) are called nodes.
- \( \bar{R}_1, \ldots, \bar{R}_n \) are relations on \( V \) (i.e. subsets of \( V^{\text{arity}(R_1)} \), \ldots, \( V^{\text{arity}(R_n)} \) respectively).

A configuration represents a hierarchical set of parameters configuring some computing equipment. The nodes representing the parameters having a name and a value.

In order to introduce the classical semantics we need the following definition.

**Definition 5.** A valuation for the above configuration is a function \( \rho : \text{Variables} \to V \)

We denote by \( \rho[x / v] \) the valuation that agrees with \( \rho \) on every variable but \( x \), in which case it returns \( v \). We will also write \( \rho(\bar{x}) \) for \( \rho(x_1) \cdots \rho(x_n) \) where \( \bar{x} = x_1 \cdots x_n \).

We can now give the classical semantics for the configuration logic.

**Definition 6.** Let \( C = < V, N, \bar{R}_1, \ldots, \bar{R}_n > \) be a configuration and \( \rho \) be a valuation for this configuration. We say that \( C, \rho \) satisfy a configuration logic formula \( \varphi \) (in notation \( C, \rho \models \varphi \)), if recursively:

- \( C, \rho \models \bar{R}_i(\bar{x}) \) if \( \bar{R}_i(\rho(\bar{x})) \) holds
3. Examples

3.1. Network Management

The global configuration of a network is formed of the configuration of its routers. To ensure proper functioning of the network, specific relations must be satisfied on the values of the parameters, which may span multiple devices.

When new network services are added, parameters of the configuration must be changed. In order to assure that all services still function properly, these changes must be made in such a way that existing relations are still fulfilled.

Due to the size of present networks and the complexity of services, it is of prime importance to develop formalisms in such a way that existing relations are still fulfilled.

3.2. Example 1: IP addresses

As has been explained earlier, the parameters of the configuration affected by a service must verify some specific relations.

The simplest example of such relation can be seen in an IP address following the Classless Inter-Domain Routing (CIDR) scheme [5], [11], whose two components, the value and the subnet mask, are linked by a simple relationship: an address like 206.13.01.48/25, having a network prefix of 25 bits, must carry a mask of at least 255.255.255.128, while the same address with a network prefix of 27 bits must not have a subnet mask under 255.255.255.224.

Figure 2 depicts a portion of a configuration representing an IP address (ip) with its subnet mask (mask) and network prefix (pref).

Let \( R(m, p) \) be a relation which holds if \( m \) is an acceptable mask for the prefix \( p \). The previous property can be expressed by the CL formula of figure 3, stating that all addresses \( a \) must have a mask \( m \) and a prefix \( p \) which satisfy \( R(m, p) \).

3.3. Example 2: Virtual Private Networks

More complex situations can be encountered, in which the parameters of several devices supporting the same service are interdependent. An example is provided by the configuration of a Virtual Private Network (VPN) service [10], [12], [13].

A VPN is a private network constructed within a public network such as a service provider’s network. A customer might have several sites geographically dispersed, and would like to link them together by a protected communication.

Most of the configuration of a VPN is realized in routers placed at the border between the client’s and the provider’s networks. On the client side, these routers are called customer edge (CE) routers, and on the provider side, they are called provider edges (PE).

Many techniques have been developed to ensure the transmission of routing information inside a VPN without making this information accessible from the outside. One frequently used method consists in using the Border Gateway Protocol (BGP). This method involves the configuration of each PE to make it a “BGP neighbor” of the other PE’s [10].

Without getting in full details, for our example it suffices to know that one interface in each PE router must have its IP address present as a BGP neighbor of each other PE router.

Let \( PE(r) \) be a relation satisfied by the PE routers, and \( neighbor(a, r) \) be a relation which holds when the address \( a \) is a BGP neighbor of router \( r \). This property can be expressed by the CL formula of figure 5, stating that for each pair of different routers \( r_1 \) and \( r_2 \) that are both PE’s, some interface of \( r_1 \) must be a BGP neighbor of \( r_2 \).
In this section, we provide a comparison of CL to other related logics.

4.1. Modal Logics

Modal ($\Diamond$, $\square$) and multi-modal ($\langle a \rangle$, $[a]$) modalities traces a path and allows to refer to properties of nodes in the future. While in modal and multi-modal logic one refers to properties of individual future states, in CL the quantifiers allow to reach different nodes and then refer to a property involving many nodes. For instance the following CL sentence

$$\langle p = x \rangle \langle p = y \rangle x \neq y$$

could at best be expressible in multi-modal logic by

$$\bigvee_{a \neq b} \langle p = a \rangle T \land \langle p = a \rangle T$$

where $a, b$ range over the domain of $x$ and $y$.

Hence classical modal and multi-modal logics can be seen as mono-site: Basic relations are on the contents of nodes. On the other hand CL can be seen as a multi-site modal logic: Basic relations can involve many nodes.

Of course the presence of variables in modalities will come at a price as we will show below.

4.2. TQL

The logic that mainly inspired the authors is the Tree Query Logic (TQL) [2, 3]. TQL has been developed as the spatial fragment of a more general logic called the ambient logic. It is a logic which not only allows formulation of sentences that can be model checked against a given tree, but also queries that extract data from those same trees. The main application of TQL is targeted towards the extraction of data from databases modeled by XML files.

Using TQL prefix operator and its quantification on arbitrary labels of nodes, one gets the CL quantifiers. Therefore, CL is a fragment of TQL.

Moreover, TQL provides fix-point operators for expressing recursive properties.

Therefore, TQL is much more expressive than CL: It allows more flexible quantifications and recursion by fix-points.

It has been shown that TQL is undecidable logic: There is no algorithm to decide if there exists a finite structure satisfying a TQL sentence [4].

We have used TQL as a tool for the validation of device configurations [7, 8]. This motivates us to investigate fragments which would be suitable to describing configurations. Our objective being to tailor a logic for configuration purpose, avoiding non necessary constructs as fix-points. Even if our logic is still undecidable as we show below, its simplicity simplify its integration in a tool. Our team is actually working on its integration in a network configuration tool.

4.3. Guarded Logics

Guarded logic is a generalization of modal logic in which all quantifiers must be relativized by atomic formulas. Therefore, quantifiers in the guarded fragment of first-order logic appear only in the form

$$\exists \bar{y}(\alpha(\bar{x}, \bar{y}, \bar{z}) \land \psi(\bar{x}, \bar{y}))$$  \hspace{1cm} (1)$$

or

$$\forall \bar{y}(\alpha(\bar{x}, \bar{y}, \bar{z}) \rightarrow \psi(\bar{x}, \bar{y}))$$  \hspace{1cm} (2)$$

The atom $\alpha$, called the guard, must contain all free variables of $\psi$ [6].

The loosely guarded logic is a generalization of guarded logic where the condition on the guard is relaxed. In this case the guard must be a conjunction of atomic formulas such that if $x$ is a free variable of $\alpha$, and $y$ is a variable from $\bar{y}$, then there is a conjunct in the guard where $x$ and $y$ both occur [9].
These fragments of first-order logic have a number of interesting properties. It has been shown [1] that the satisfiability problem for the guarded fragment is decidable, and, moreover, that it has the finite model property (every satisfiable formula in the guarded fragment has a finite model). The loosely guarded fragment has been shown to have the small model property [9].

Unfortunately CL configuration properties are neither guarded nor loosely guarded. For instance if we consider the subformula

\[(\text{router} = r_1; \text{inter} = i)\text{Neighbor}(i,r_2)\]

of the sentence of figure 5, which can be translated in first-order terms to

\[\exists_i I(r_1, i) \land \text{Neighbor}(i, r_2)\]

where \(I(r, i)\) holds if \(i\) is a interface of router \(r\), then \(r_2\) is a free variable of \(\text{Neighbor}(i, r_2)\) which is not in the guard \(I(r_1, i)\).

Furthermore there is in general for CL sentences no equivalent guarded or loosely guarded equivalent sentences. This follows from the fact that one can define an infinite total order in CL by the following sentences on one binary relation \(R\). The conjunction of these sentence is consistent, but it has no finite model, hence the finite model property does hold for configuration logic.

\[\lnot R(x, x)\]
\[\lnot R(x, y) \land R(y, z) \rightarrow R(x, z)\]
\[\lnot R(x, y)\]

4.4. From classical first-order logic to CL

In fact classical first-order logic can be interpreted in CL by replacing existential quantifiers \(\exists x\) by \((p = x)\) and universal quantifiers \(\forall x\) by \([p = x]\), for some fix name \(p\).

By Trakhtenbrot’s result [14] which states that for a first-order language including a relation symbol that is not unary, satisfiability over finite structures is undecidable, we have the analogue for CL.

Therefore there can be no effective way to find a bound on the size of the smallest finite model of a CL formula, since enumerating the structure of this size would give decidability for the existence of a finite structure satisfying the sentence.

5. Adapted Valuations

As shown above CL doesn’t have nice computational properties like decidability and small model property. We are investigating fragments of CL which would be expressive enough for the application we have in mind and which would have these property. In this paper we make a first step in investigating CL, by giving a semantics in terms of partial functions on variables.

We already gave a classical semantics for our formalism. Even if this semantics is satisfactory, we think that since our logic is about paths in trees it is interesting to propose a new semantics, equivalent to the previous, but based on the path and tree structure of the formula.

The idea is that in order to check a sentence one has to recursively check sub-formulas. To check a sub-formula one has to consider valuations. We show in this section that instead of considering general valuations, it is sufficient to restrict ourself to functions sending variables to values which satisfies the hierarchical structure of variables in the sentence. This allows to integrate the hierarchical condition on the values of variable in the definition of these new kind of valuation.

We propose this new semantics in this section and show that it is equivalent to the previous classical semantics.

**Definition 7.** The path of a sub-formula of the form \((p = x)\psi\) or \((p = x)\psi\) is \((p = x)\), the path of a sub-formula \((\bar p = \bar x; q = y)\psi\) and \((\bar p = \bar x; q = y)\psi\) is \((\bar p = \bar x)(q = y)\).

Since in a specific sentence a variable is bounded only once, we will speak of the path of a bounded variable which is the path of the quantifier binding this variable in the sentence.

From definition 1 one can show by induction that the following result holds.

**Proposition 1.** Let \(\varphi\) be a sentence and \(x\) a variable of \(\varphi\) of path \((p_1 = x_1), \ldots, (p_n, x_n)(p, x)\). For all \(i = 1, \ldots, n\) we have that the path of \(x_i\) is \((p_n = x_1), \ldots, (p_i, x_i)\).

We will say that \(f : A \rightarrow B\) is a partial function if \(f\) is a function sending elements of its domain \(\text{dom}(f)\) to elements of \(B\).

Let \(\varphi\) be a CL formula, we will denote by \(\text{Variables}(\varphi)\) the set of variables (bound or free) of \(\varphi\).

We can now give the definition of our restricted form of valuation.

**Definition 8.** Let \(C = \ll V, N, \tilde{R}_1, \ldots, \tilde{R}_n \gg\) be a configuration and \(\varphi\) be a sentence. A partial function \(\rho : \text{Variables}(\varphi) \rightarrow V\) is said to be adapted (or \(\varphi\)-adapted) for \(C\) if for every variable \(y \in \text{dom}(\varphi)\) of \(\varphi\) of path \((p_1 = y_1), \ldots, (p_n = y_n)(p = y)\), the following conditions holds:

1. \(\{y_1, \ldots, y_n\} \subseteq \text{dom}(\rho)\)
2. \((p_1 = \rho(y_1)), \ldots, (p_n = \rho(y_n))(p = \rho(y)) \in N\).
We now have the following fact.

**Proposition 2.** Let \( C = \langle V, N, \tilde{R}_1, \ldots, \tilde{R}_n \rangle \) be a configuration, \( \varphi \) be a sentence, and \( \rho \) be a valuation for \( C \) adapted to \( \varphi \).

Let also \((p_1 = x_1) \cdots (p_r = x_r)(q = y)\) be the path of \( y \) in \( \varphi \) for some \( y \notin \text{dom}(\rho) \).

We have that if \( \{x_1, \ldots, x_n\} \in \text{dom}(\rho) \) and if \( v \in V \) is such that \((p_1 = \rho(x_1)) \cdots (p_r = \rho(x_r))(q = v) \in N \) then \( \rho' = \rho[y/v] \) is adapted to \( \varphi \).

**Proof.** To prove that \( \rho' = \rho[y/v] \) is adapted, we must show that for \( y' \in \text{dom}(\rho') \) of path \((q_1 = y_1) \cdots (q_m = y_m)(q = y') \) it holds that

1. \( \{y_1, \ldots, y_n\} \subseteq \text{dom}(\rho') \)
2. \( (q_1 = \rho'(y_1)) \cdots (q_m = \rho'(y_m))(q = \rho'(y')) \in N \)

As \( y \notin \text{dom}(\rho) \), \( y \) cannot appears in any path of \( \text{dom}(\rho') \) except its own. Therefore the claim must be shown only for \( y' = y \).

For \( y = y' \) the claim follows from the hypothesis. \( \square \)

By the previous result we have that if \( \rho \) of definition 6 is adapted and if its domain contains all free variable of the formula under consideration but not \( y \) then the valuations \( \rho[y/v] \) considered in this definition are again adapted.

This fact make it possible to make more precise the relationship between adapted and normal valuations.

**Lemma 1.** Let \( \varphi \) be a sub-formula of some sentence \( \varphi' \), let \( C = \langle V, N, \tilde{R}_1, \ldots, \tilde{R}_n \rangle \) be a configuration and \( \rho \) be a valuation for \( C \) whose domain contains all free variables of \( \varphi \).

If \( F \) is a set of variables containing all free variables of \( \varphi \) but none of its bounded variables and if \( \rho[F] \), the restriction of \( \rho \) to the domain \( F \), is \( \varphi' \)-adapted then the following conditions are equivalent

1. \( C, \rho \models \varphi \)
2. \( C, \rho[F] \models \varphi \)

**Proof.** The proof goes by induction on the structure of \( \varphi \).

The case of an atomic formula, conjunction, disjunction and negation is clear.

All cases of existential and universal quantifiers are similar, so we give details only for \( \varphi = (\bar{p} = \bar{x}; q = y)\psi \).

If \( C, \rho \models (\bar{p} = \bar{x}; q = y)\psi \), then by definition 6, we have that there exists a \( v \in V \) such that \((\bar{p} = \rho(\bar{x}))(q = v) \in N \) and \( C, \rho[y/v] \models \psi \).

Now since \( y \notin F \) it follows that \( y \notin \text{dom}(\rho[F]) \). Therefore by Proposition 2 it follows that \( (\rho[F])[y/v] \) is \( \varphi' \)-adapted.

Let \( F' = F \cup \{y\} \). We have that \( (\rho[F])[y/v] = \rho[y/v][F'] \), since they both agree on \( F \) and on \( y \). So \( \rho[y/v][F'] \) is \( \varphi' \)-adapted.

Since \( F' \) contains all free and no bounded variable of \( \psi \), it follows by induction hypothesis that \( C, \rho[y/v][F'] \models \psi \) holds. Again by equality \( \rho[F][y/v] = \rho[y/v][F'] \), and by definition 6 we have that \( C, \rho[F][y/v] \models (\bar{p} = \bar{x}; q = y)\psi \) holds.

Conversely if \( C, \rho[F][y/v] \models (\bar{p} = \bar{x}; q = y)\psi \), by definition 6, we have that there exists a \( v \in V \) such that \((\bar{p} = \rho(\bar{x}))(q = v) \in N \) and \( C, \rho[F][y/v] \models \psi \).

As before, it follows from Proposition 2 that \((\rho[F])[y/v] \) is \( \varphi' \)-adapted. Furthermore \((\rho[F])[y/v] = \rho[y/v][F'] \), holds.

Therefore we have by induction hypothesis that \( C, \rho[y/v] \models \psi \) and hence \( C, \rho \models (\bar{p} = \bar{x}; q = y)\psi \). \( \square \)

It now follows that:

**Theorem 1.** Let \( C = \langle V, N, \tilde{R}_1, \ldots, \tilde{R}_n \rangle \) be a configuration, \( \varphi \) be a sentence and \( \rho \) be a valuation for \( C \). Let \( \emptyset \) be the empty \( \varphi \)-adapted valuation (its domain is the empty set). We have that \( C, \rho \models \varphi \) if and only if \( C, \emptyset \models \varphi \).

From the previous result we get the following equivalence.

**Theorem 2.** Let \( C = \langle V, N, \tilde{R}_1, \ldots, \tilde{R}_n \rangle \) be a configuration and \( \varphi \) be a sentence. The following condition are equivalent.

1. \( C, \rho \models \varphi \) for all valuations \( \rho \)
2. \( C, \rho \models \varphi \) for some valuation \( \rho \)
3. \( C, \rho \models \varphi \) for some \( \varphi \)-adapted valuation \( \rho \)
4. \( C, \rho \models \varphi \) for all \( \varphi \)-adapted valuations \( \rho \)

**Proof.** Follows directly from Theorem 1 since to check that \( C, \rho \models \varphi \) for some valuation \( \rho \) it suffice to check that \( C, \emptyset \models \varphi \) holds. \( \square \)

**Remark 1.** It is important to note that the hierarchical structure of variables contain the possible adapted valuations. Therefore even if the empty valuation is always an adapted valuation, there is not always an adapted valuation whose domain contains all free variables as shown by the following example.

**Example 1.** If \( C = \langle V, N, \tilde{R}_1, \ldots, \tilde{R}_n \rangle \) is such that \( N \) contains no \( \bar{p} = v \) for some name \( p \) then there is no \( \varphi \)-adapted valuation on \( C \) for \( \varphi = (\bar{p} = x)x = x \), whose domain contains \( x \).
6. Conclusion

We proposed a new logic for describing configuration of computing equipments and motivated it with examples from network configuration. We also gave a classical and a new equivalent semantics.

Since we are interested in application, we are working at integrating CL in a network configuration tool. We are also working on using our new semantics to investigate fragment of CL which would be sufficient to express the properties needed in practice, while having better theoretical properties like decidability and small model property.

References


