THE PARADOXICAL TWINS

As Professor Samson approached his house after a hard day in the laboratory he heard a depressingly familiar sound. "My God!" he groaned to himself, "Why did I have to be the one father out of a hundred who is cursed with twins?" With sinking heart he opened his front door and bellowed, "Grace! Gerald! Stop that racket this instant!" Having lowered the decibel level sufficiently to be heard speaking at something approaching a normal tone of voice, he continued, "You're almost fourteen and you're still fighting like a couple of little kids. I buy you a popular science book so that you'll be too busy to fight and you fight over who gets to read the book. I buy you a second copy and you're still fighting. What are you fighting about now?"

"About something it says in the book!" replied Grace. "Listen to this. It's in the chapter called The Twin Paradox - about the twin who voyages away from the Earth at close to the speed of light and then returns and compares his age with that of his stay-at-home brother. It says, 'Each twin sees the other moving relatively to himself; so each twin sees the other one's watch as going slower than his own, and each could assume with equal justification that his brother will age less than he will.' Notice, by the way, that the author assumes that both twins are boys! 'But the twin who makes the two-way trip is accelerating and the twin who stays at home is not. Since the Special Theory of Relativity says nothing about accelerated motion, it is necessary to turn to the General Theory to decide which twin will age more.'"

"Of course!" scoffed Gerald. "The General Theory says that time slows down when a body accelerates; so the twin who voyages will age less."

"But suppose the voyage is so long that the acceleration time is far shorter than the time spent in uniform motion?" Grace persisted. "If the Special Theory predicts that one twin will age more than the other, then this effect can be made arbitrarily large compared to any slow-down predicted by the General Theory; so it should be possible to design an experiment to see what the Special Theory alone has to say on the matter."

"Well, go ahead and design one, Miss Know-It-All!" sneered Gerald.

"That has already been done," Professor Samson cut in quickly, anxious to preclude any resumption of hostilities. "Rather than making the voyage long, you make the acceleration time short by using particles that are small enough to stand up to enormous g-forces. You two have pummeled each other so much that you can take as many g's as a quark. And I've designed a space ship that will enable you to repeat the experiment for yourselves. Are you game?"

The question, of course, was rhetorical. Professor Samson was delighted by his children’s enthusiasm - until they began arguing over which of them was gamer.

On the next clear night Professor Samson drove his two children to his private launching pad. On the pad there was a spaceship labelled Einstein. As soon as the car stopped, both children raced for the ship. When their father finally reached the ship, he found them fighting for space inside its tiny interior.

"There's only room for one of you in the ship," he said, and then, anticipating another quarrel, he added quickly, "but don't worry: each of you will get to make one trip."

Both children immediately began shouting "Me first!" until he flipped a switch, causing two loudspeakers, one on the ship and the other on the ground nearby, to begin emitting synchronous ear-splitting beeps.

As he had hoped, the two children were startled into momentary silence. Before they could resume their current quarrel, he began to deliver them a scientific lecture, for this was the thing he most enjoyed doing. "I'm sure you've both noticed that a train-whistle seems to get lower in pitch as the train passes..."

"Sure, sure, the Doppler effect, I've read all about it," Gerald interrupted.

"...and that a stationary bell also appears to a passenger on the train to get lower in pitch as the train passes the bell," continued Professor Samson. "Now, would the bell also appear to ring less often after the train passes it than before?"

"Of course!" shouted Gerald again. "While the train is approaching the bell, each time it rings it's closer to the passenger; so it takes less time for the sound to get to the passenger and he hears it ring more often than the train-spotter standing by the bell. After the train passes the bell, it's farther away each time it rings; so the sound takes longer to reach the passenger and he hears it ring less often. If the whistle were tooting in rhythm, the train-spotter would hear it toot faster than the passenger while the train was approaching and slower after it passed."

"And now suppose the bell and the whistle have been programmed to go at the same speed," continued the professor. "Which will sound slower after the train has passed, the bell to the passenger or the whistle to the train-spotter?"

Both children paused briefly, allowing their father to enjoy the relative calm of the beeping loudspeakers. "The bell!" shouted Grace. "Suppose the train were moving at the speed of sound. The dings from the bell would never get to the passenger, but the toots from the whistle would still get to the train-spotter; so moving away from the source of sound slows things down more than having the source moving away from you at the same speed."

"Good!" beamed their father. "And which will sound faster before the train passes, the..."

"The whistle!" shouted Gerald, anxious to beat his sister to the punch. "If the train were moving at the speed of sound, all the toots from the whistle would get to the train-spotter at the same time as the train - a sort of sonic toot - whereas the dings from the bell would still get to the passenger one at a time; so having the sound-source moving towards you speeds things up more than moving towards the source at the same speed."

"Good!" beamed their father again, careful to express exactly the same degree of enthusiasm for both children to avoid another quarrel. "So in either case, the whistle toots faster to the train-spotter than the bell dings to the passenger. Right?" Both children nodded simultaneously. Glad that they had found something on which they could agree, he continued, "Now, suppose the train goes a mile down the track and then stops, goes backwards and passes the train-spotter again in the other direction, and all that time both the whistle and the bell are sounding. The train-spotter is hearing the toots from the whistle more often than the passenger is hearing dings from the bell, both when the train is receding and when it is approaching. Does this mean that from the first time the train passes the train-spotter until the second time it passes in the other direction the train-spotter hears more noises than the passenger?"

"Of course not!" scoffed Gerald. "The whistle and the bell are making noises at the same frequency and for the same amount of time; so the train-spotter has to hear the same number of noises from the whistle as the passenger hears from the bell."

"Of course," said their father slyly, "but how could that be if the train-spotter is always hearing noises more frequently than the passenger?"

Both children raced to think of the explanation for the apparent contradiction. This particular race was won by Grace: "The passenger stops hearing slow dings and starts hearing fast dings as soon as the train starts going backwards; so she hears slow and fast dings for the same amount of time. But until the toot made by the whistle at the moment the train changes direction reaches the train-spotter, he keeps hearing slow toots; so he hears slow toots for a long time and fast ones for a short time. That makes up for the fact that he hears faster noises than the passenger in both cases."

The father continued his demonstration in his most professorial tone. "Now suppose we replace both the bell and the whistle by lights flashing at the same frequency. While the train is receding, both the passenger and the train-spotter will see the other one's light flashing slower. Will the passenger see slower flashes than the train-spotter?"

"No!" shouted Gerald. "If they were in space-ships instead of a train and the Earth, they could still tell which one was moving by timing the other's flashes and then comparing notes, and the Special Theory says that's impossible. They each see the other's light flashing at the same speed both when they're receding and when they're approaching."

The professor, expecting to have to ask several more questions, continued, "Does the passenger still see slow flashes and fast ones for the same amount of time and does the train-spotter still see slow flashes longer than fast ones?"

"Of course!" shouted Grace. "So the passenger sees more flashes than the train-spotter; so the light by the tracks has to make more flashes than the light on the train. That means that the train-spotter ages more than the passenger!"

"You're 'way ahead of me!" beamed her father.

Gerald too was 'way ahead of his father. "Hey! I'll bet that's what the beeping is for! The loudspeaker on the space-ship is for the whistle and the one on the ground is for the bell. And I'll bet you've got flashing lights rigged up too! We're gonna play passenger and train-spotter, Sis, and I'm the passenger!"

Before Grace could respond, her father raised his hand and said: "I already told you that you'll each get to make one trip. One of you gets into the spaceship, and I will launch it at four-fifths of the speed of sound to the next town. It takes sound four minutes to get from one town to the other; so it'll take the ship five minutes. As soon as it arrives, another device I rigged up will bounce it back here at the same speed. During both legs of the journey, the observer in the ship will count the beeps coming every minute from the loudspeaker on the ground and the observer on the ground will count the beeps coming every minute from the spaceship. Then you change places, except that the beeps will be exchanged for flashes of light and the trip will be to Mars at four-fifths of the speed of light. Mars is at favourable opposition now; so it takes light four minutes to get there, and the ship will get there in five. Now which of you wants..."

"I get to go to Mars!" shouted both twins simultaneously, and then immediately started fighting. Their father stepped between them, dodging blows from both sides as he said, "There is a civilized way to decide this issue!"

"You're darned right there is!" Gerald proclaimed. "I get to go to Mars because I'm older!"

"Yeah, three minutes older!" sneered his sister.

"So? I'm still older!" replied Gerald.

"I'm afraid he's got you there, Grace," said Professor Samson, anxious to have the argument settled one way or the other.

"You men are all alike!" Grace grumbled with an air of grudging resignation.

Relieved that one more argument had been stopped and the next one hadn't yet begun, Professor Samson began didactically: "Both loudspeakers are beeping forty-five times a minute. You each use your watch to time the frequency at which you hear the other one's loudspeaker beeping during each leg of the journey and the length of time you see each leg taking. Grace, climb into the spaceship and then I'll start the countdown at forty-five beeps."

At the count of zero Professor Samson pressed a button. The spaceship took off as suddenly as if it had been shot from a cannon, and soon it disappeared over the horizon. One minute later, Professor Samson asked his son, "Okay, Gerald, how often do you hear Grace's loudspeaker beeping?"

"Twenty-five times a minute," replied Gerald.

"Can you explain why?"

"Of course!" boasted Gerald. "When she gets to the next town she will have been travelling for five minutes. At forty-five beeps a minute, her speaker will have emitted two hundred and twenty-five beeps. The last of those beeps will take four minutes to get to me - that's nine minutes after she started. So it'll take me nine minutes to receive those two hundred and twenty-five beeps, and that's twenty-five beeps a minute."

"Good!" beamed his father. "Now, Grace, how often do you hear Gerald's loudspeaker beeping?"

"Nine times a minute," came her voice from the monitor. "When I get to the next town, I'll hear the beep Gerald's speaker emitted four minutes earlier - that's one minute after I started. During that one minute his speaker emitted forty-five beeps, but it'll take me five minutes to receive them, and that's nine beeps a minute."

"Excellent!" exclaimed her father, glowing with pride. "As soon as you get bounced back, start counting the new rate at which you hear Gerald's loudspeaker beeping. As you, Gerald, as soon as you hear the ship being bounced back, start counting how often you hear Grace's loudspeaker beeping."

Soon the air was rent by a sound resembling the crack of a baseball bat. A minute later the spaceship reappeared and landed in a net right beside the launching pad. "Two hundred and twenty-five beeps a minute," announced Gerald. "It took her five minutes to get back, and during that time her speaker emitted two hundred and twenty-five beeps. But the first of them took four minutes to get to me; so I received it only one minute ago. And I received all those two hundred and twenty-five beeps in a single minute."

Emerging from the spaceship, Grace trumpeted. "And I heard eighty-one beeps a minute after I got bounced back. The beep I heard when I got bounced back had been emitted four minutes earlier - that's nine minutes before I landed. So I heard a total of nine times forty-five, or four hundred and five, beeps during the five minutes it took me to get back, and that's eighty-one beeps a minute."

"Good! Now let's summarize these results in a diagram and a table," said Professor Samson, producing the pen and paper without which no professor ever appears in public and drawing the diagram and the table which appear as Figure 1 and Table 1 on the second last page of this story. Then, recalling having been advised to repeat the salient points of his lectures, he continued: "Not surprisingly, you each heard the same number of beeps from the other; so you each emitted the same number of beeps and aged the same amount. Now tell me: suppose you were receding from each other at four-fifths the speed of sound and neither of you had been accelerated. How could you tell which of you was moving and which was standing still?"

"That's easy!" said Gerald. "We communicate with each other and compare how often we hear each other's beeps. I hear hers at twenty-five a minute and she hears mine at nine; so she's the one who's moving."

"And if you're approaching each other at four-fifths the speed of sound?"

"I hear hers at two hundred and twenty-five a minute and she hears mine at eighty-one; so she's the one who's moving."

"So whether we're approaching or receding, the one who hears the slower beeps is moving!" said Grace, glad to have made a profound generalization.

"Now tell me this," pontificated Professor Samson. "During both legs of the journey, the voyager hears beeps less often than the stay-at-home. So how come they both hear the same total number of beeps?"

The twins raced to finish their answer on the assumption that whoever finished talking first would get the credit: "Because the stay-at-home hears slow beeps for nine minutes and fast ones for only one and the voyager hears slow beeps for five minutes and fast ones for five minutes."

"Now suppose you went at four-fifths the speed of light and watched light-flashes instead of listening to beeps," continued Professor Samson. "Would the stay-at-home still see slow flashes for a long time and fast ones for a short time and the voyager see fast ones and slow ones for the same amount of time?"

"Of course!" answered Gerald. "The voyager stops seeing slow flashes and starts seeing fast ones as soon as he turns around, but the stay-at-home has to wait until the light sent by the voyager at the moment he turns around gets to her before she stops seeing slow flashes and starts seeing fast ones."

"Would the voyager still see flashes less often than the stay-at-home during both legs of the journey?" asked the father.

"Of course not!" answered Grace. "Otherwise they could tell which one was moving and the Special Theory says that's not possible."

"Well, then," concluded the professor triumphantly, "which one is going to see more of the other one's flashes?"

"The voyager!" answered both twins simultaneously.

"If the voyager sees more flashes, then the stay-at-home has to emit more flashes. That means that the stay-at-home ages more than the voyager," Grace deduced, and then added scornfully, "And you don't need the General Theory to figure that one out either, Mr. Know-It-All!"

"I'll believe it when I see it," grumbled Gerald, unwilling to admit defeat.

"Synchronize your watch with your sister's and then climb into the spaceship and see for yourself," said his father quickly, anxious to avoid another quarrel. "My spaceship-bouncer on Mars has signaled that it’s ready to do its stuff."

A few seconds after liftoff, all that could be seen of the spaceship was the flashing light. Professor Samson waited a minute and then asked "How often do you see the flashes, Grace?"

"Fifteen times a minute instead of twenty-five a minute like before; so his flasher must have slowed down to three-fifths of its normal speed."

"That's what you've deduced assuming that you were standing still in some sort of absolute space and he were moving in it," added her father.

"And I see Grace's flashes at fifteen a minute as well," came Gerald's voice over the monitor. "But if there's no way of deciding which of us is moving, I can just as easily say that she's doing the moving and so I ought to be seeing her flashes at twenty-five a minute and it's her flasher that's slowed down!"

"Right!" said his father. "That's what the word Relativity means: everything depends on your point of view. I wish you two could come to the same conclusion instead of fighting all the time."

"But we could just as easily see each other's flashes at twenty-five a minute or nine or seventeen or any other number and we still wouldn't know which of us was moving," Grace protested. "What's so special about fifteen?"

"I'll explain that to you both as soon as your brother returns," replied her father. "Meanwhile, as soon as you see the bright flash made by the ship being bounced back, tell me how much time has elapsed since Gerald departed. And you, Gerald, as soon as you get bounced back, tell me how much time you've been travelling."

Immediately after the bright flash, the twins' voices piped up simultaneously.

"Nine minutes," announced Grace.

"Three minutes," announced Gerald, and then he added, "Hey! She said nine minutes! So according to her, it took five minutes for me to get here and four minutes for the signal that I'd got here to get back to her. How come it only took me three minutes to get here? Is it because the distance from Earth to Mars looks shorter when you're moving relative to the planets?"

"The other way around," replied his father. "She saw your flashes at fifteen a minute for nine minutes; so you sent her one hundred and thirty-five flashes. You've been sending them at forty-five a minute; so it took you three minutes by your watch to get to Mars instead of five. Since your speed is still four-fifths of the speed of light, that makes the distance from Earth to Mars seem shorter to you than it does to us. It's not as if you really saw Mars as being any closer. Now, both of you measure the frequency at which you see each other's flashes and the time taken for the return journey."

"One hundred and thirty-five a minute for one minute," said Grace as soon as the spaceship landed in the net.

"One hundred and thirty-five flashes a minute for three minutes," said Gerald as soon as he emerged from the spaceship. "And now I know what it means that we each see the other's watch as going slower than our own. We don't always see the other one's flashes as slower than our own;

when we're approaching each other we see them faster. But we each see the other one's flashes coming slower than if we were standing still in absolute space and the other were moving in it. For instance, when we were approaching each other we saw each other's flashes at one hundred and thirty-five a minute instead of two hundred and twenty-five! So we each deduce that the other one's flasher has slowed down, just like I deduced that the distance from Earth to Mars had shortened because it took me less time to get there than I expected, and if we could see each other's watch we'd figure that it had slowed down too!"

"Exactly! Some popular science books over-simplify the things they talk about, leaving a wrong impression, and it takes a professor to set the record straight," boasted his father. "Now let's make another diagram and table." With that, he produced another piece of paper and drew Figure 2 and Table 2.

"You see," he continued, "at the rate of forty-five flashes a minute, it took Grace one minute to get to flash number forty-five, and four minutes more for that flash to get to Mars; so it got there five minutes after you left - the same time you did. If your watch had advanced five minutes, you would have seen her forty-five flashes at the rate of nine a minute. And you would have sent out two hundred and twenty-five flashes which she would have received in nine minutes - that's twenty-five a minute, much more frequently than you'd have received hers. But your watch advanced only three minutes instead of five. So you saw her forty-five flashes at fifteen a minute instead of nine. And you emitted only one hundred and thirty-five flashes, which she received in nine minutes - that's also fifteen a minute instead of twenty-five. The slower your watch goes, the more frequently you see her flashes and the less frequently she sees yours, and the only way you can each see the other's flashes at the same frequency is for your watch to advance exactly three minutes.

"The same thing happened during the return journey. When you got bounced back, you received the flash she emitted one minute after you started out; so during the return journey you received the flashes she emitted during nine minutes. But the flash you emitted when you got bounced back took four minutes to get to her and you took five minutes; so it took her only one minute to receive all the flashes you emitted during your return journey. Your watch advanced three minutes. So you received the four hundred and five flashes she emitted in nine minutes at the rate of one hundred and thirty-five a minute. And you emitted one hundred and thirty-five flashes which she received in one minute - the same rate as you received hers. Altogether, then, your watch advanced six minutes while hers advanced ten. Now compare your watches and you'll find that yours is four minutes behind hers."

The twins complied and nodded their agreement.

"That means that she aged ten minutes and you only six," he concluded triumphantly, "and General Relativity had nothing to do with it!"

Grace stuck out her tongue and chanted, "I told you so! I told you so!"

Gerald lunged at his sister, but their father stepped between them and shouted, "Hey! There are more interesting things to do than fight with each other now that you've both proved your ability to make such fuel-efficient journeys into space. For instance, NASA wants to conduct a manned expedition to Pluto and ..."

"He said a manned expedition; so that leaves you out, Sis!" sneered Gerald.

Grace uttered a string of unprintable epithets which can be loosely translated as 'male chauvinist pig' until she was interrupted by her father: "And I've just received word that there's an advanced civilization on one of the planets orbiting Sirius, and they want someone from Earth to come and visit them."

"As you were saying, Gerald, that leaves me out of the Pluto trip!" crowed Grace.

"Nothing doing!" retorted Gerald. "I get to go to Sirius 'cause I'm older than you, remember?"

"Not any more, you're not!" sneered Grace. "You were three minutes older than I was, but I aged four minutes more than you did during your trip to Mars; so now I'm one minute older than you are! Make sure to bring lots of bones to feed to Pluto!"

"I'm afraid she's got you there, Gerald," said Professor Samson. "But don't worry: you'll be able to take lots of interesting trips while she's away. And when she gets back she'll be several years younger than you; so you'll have first crack at the next trip that comes up."

Both children were happy when they left on their respective trips, but the happiest of all was their father. It would be a long time before he'd have to referee another fight between his children!

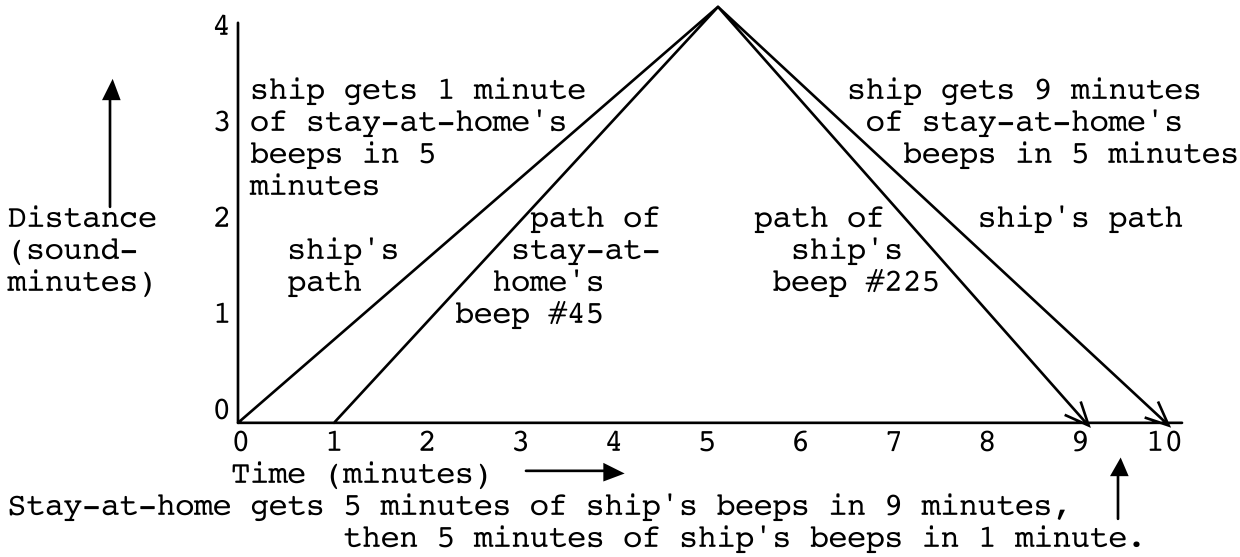

FIGURE 1: NON-RELATIVISTIC DOPPLER EFFECT

TABLE 1: NON-RELATIVISTIC DOPPLER EFFECT

OBSERVER:	stay-at-home	voyager
beeps emitted per minute:	45	45
VOYAGER RECEDING AT 4/5 THE SPEED OF SOUND		
beeps heard per minute:	25	9
time taken (minutes):	9	5
total beeps heard:	225	45
VOYAGER APPROACHING AT 4/5 THE SPEED OF SOUND		
beeps heard per minute:	225	81
time taken (minutes):	1	5
total beeps heard:	225	405
TOTALS		
time taken (minutes):	10	10
other observer's beeps heard:	450	450

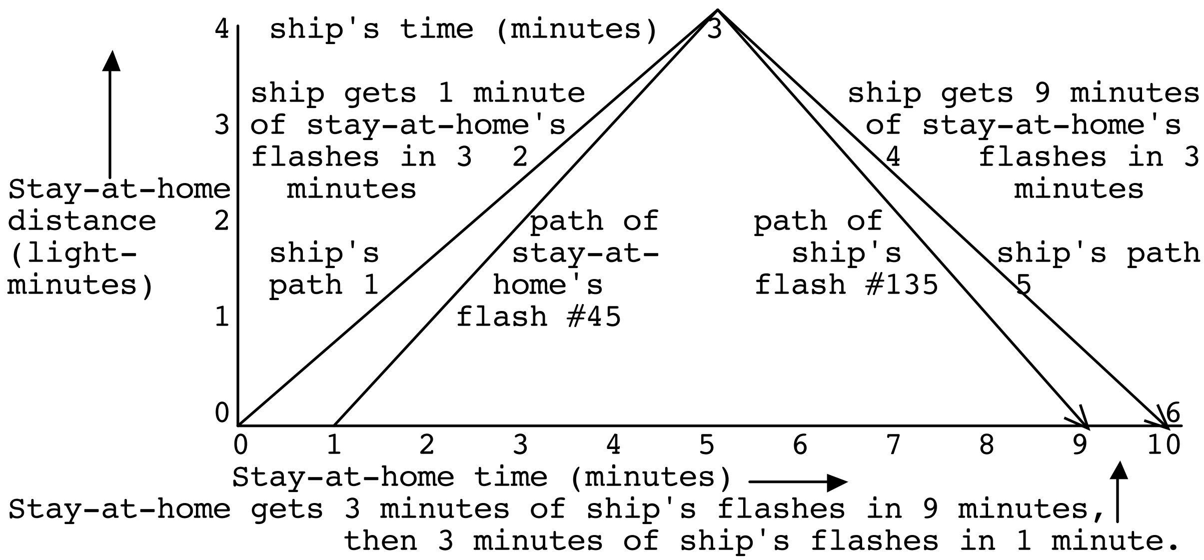

FIGURE 2: RELATIVISTIC DOPPLER EFFECT

TABLE 2: RELATIVISTIC DOPPLER EFFECT

OBSERVER:	stay-at-home	voyager
flashes emitted per minute:	45	45
OBSERVERS RECEDING FROM EACH OTHER AT 4/5 THE SPEED OF LIGHT		
flashes seen per minute:	15	15
time taken (minutes):	9	3
total flashes seen:	135	45
OBSERVERS APPROACHING EACH OTHER AT 4/5 THE SPEED OF LIGHT		
flashes seen per minute:	135	135
time taken (minutes):	1	3
total flashes seen:	135	405
time taken (minutes):	10	6
other observer's flashes seen:	270	450